
CHAPTER 6

**PERMEABILITY
OF
SOIL**

تتحرك المياه الحرة داخل وخارج التربة نظرا لأن فراغاتها البينية متصلة . وتختلف قدرة التربة في إنفاذ المياه باختلاف الظروف المحيطة وتعتمد على خواص التربة وأهمها تدرجها . ودراسة نفاذية التربة لها تطبيقات عديدة من أهمها التسرب أسفل المنشآت وما يمكن ان يسببه من مشكلات بالنسبة لآثارها , وإتزان القاع لحفر تم تنفيذه أسفل المياه الجوفية وتم خفض المياه داخله , وحساب تصرفات الآبار وخفض المياه الجوفية المناظر لذلك , وتطبيقات عديدة أخرى .

وتختلف قيمة معامل نفاذية التربة إختلافا كبيرا , فنفاذية الطمي تقل عن نفاذية الرمل بعشرات المرات , ونفاذية الرمل (النظيف) المتدرج مائة مرة نفاذية الرمل الطمي . وتقل النفاذية للطين بمئات المرات عن نفاذية الرمل , والطين المتماسك غير منفذ عمليا . والواقع أن معامل نفاذية التربة الحقيقي في الموقع من أكثر المعاملات حساسية وصعوبة في التحديد سواء بالتجارب المعملية او الحقلية .

6.1 Definition

Permeability is defined as the property of a porous material which permits the seepage of water (or other fluids) through its connected voids. Gravels are highly permeable while stiff clay is practically impermeable.

The study of seepage of water through soils is important for the following engineering problems: Fig. 6.1 (a), (b)

- (a) Calculation of seepage through the body of earth dams, and stability of slopes.
- (b) Groundwater flow towards wells and drainage of soils.
- (c) Calculation of uplift pressure on underground structures and safety of excavations against piping.

- (d) Determination of the rate of settlement of a saturated compressible soil.



Fig. 1(a) Use of well points to lower groundwater table in soil for excavation

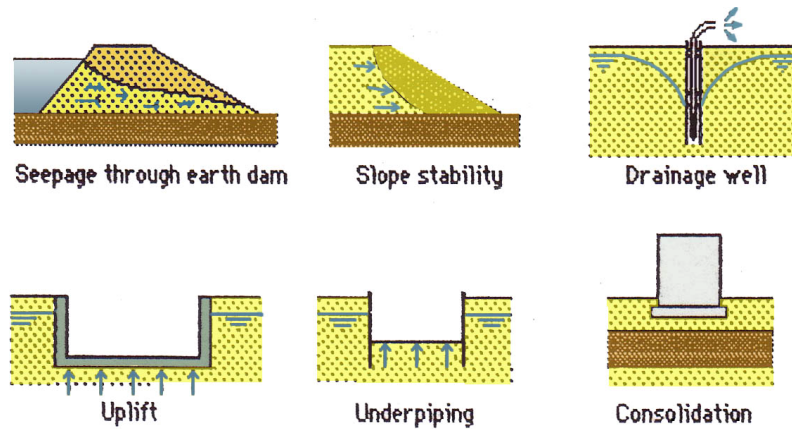


Fig. 1(b) Examples of using permeability property in soil problems

6.2 Darcy's Law

It was found experimentally that for laminar flow of water through soil the rate of flow, i.e. discharge per unit time, is proportional to the hydraulic gradient, i.e.:

$$q = k \cdot i \cdot A \quad \dots \dots \dots (6.1)$$

$$\text{or : } v = k \cdot i \quad \dots \dots \dots (6.2)$$

where:

q = discharge per unit time.

A = total cross sectional area of soil mass Perpendicular to the direction of flow.

i = hydraulic gradient.

k = Darcy's coefficient of permeability.

v = velocity of flow, or average discharge velocity.

If a soil mass of length (L) and cross sectional area (A), Fig. 6.2, is subjected to differential head of water $h = h_1 - h_2$, the hydraulic gradient (i) will be equal to h/L . Then:

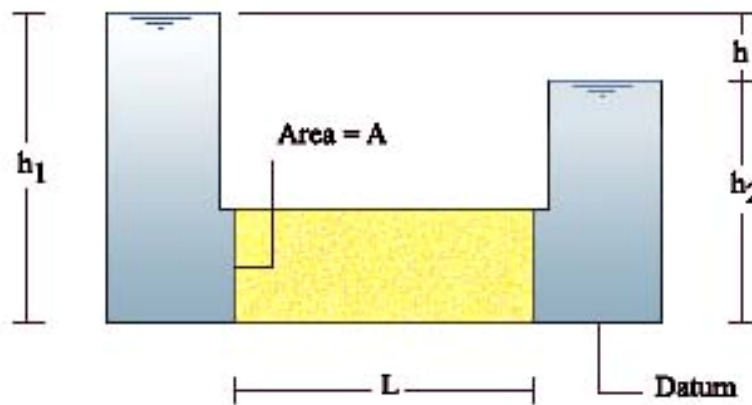


Fig. 6.2 Presentation of Darcy's law

$$q = k \cdot \frac{h}{L} \cdot A \quad \dots \dots \dots (6.3)$$

It is noticed that the dimension of the coefficient of permeability (k) are the same as those of velocity, usually expressed as cm/sec, or m/day.

Typical values of the coefficient of permeability of various soils are given in Table 6.1.

Table 6.1 Typical values of soil permeability

Soil Type	Coefficient of permeability (cm/sec)	
Clean gravel	> 1	
Clean coarse sand	1	to 1×10^{-2}
Sand	1×10^{-2}	to 5×10^{-3}
Fine sand	5×10^{-2}	to 1×10^{-3}
Silty sand	2×10^{-3}	to 1×10^{-4}
Silt	5×10^{-4}	to 1×10^{-5}
Clay	< 1×10^{-6}	

6.3 Discharge velocity and seepage velocity

The velocity of flow (v) is the rate of water per unit of the total cross sectional area (A) of soil. This area is composed of area of solids (A_s) and area of voids (A_v). Since the flow occurs through the voids, the actual velocity of flow will be greater than the discharge velocity. This actual velocity is called seepage velocity (v_s), and is defined as the rate of discharge of percolating water per unit cross sectional area of voids perpendicular to the direction of flow. Therefore:

$$q = v \cdot A = v_s \cdot A_s \quad \therefore v_s = \frac{v}{n} \quad \dots \dots \dots (6.4)$$

where: n = porosity.

The seepage velocity is also proportional to the hydraulic gradient, i.e.

$$v_s = k_p \cdot i \quad \dots \dots \dots (6.5)$$

where: k_p = coefficient of percolation.

6.4 Laboratory determination of the coefficient of permeability

الطرق المعملية لتعيين معامل النفاذية

يناسب اختبار الضاغط الثابت التربة الخشنة إذ يلزم لتعيين معامل النفاذية الحصول على حجم مناسب من المياه في زمن مناسب ، كما هو واضح من أسلوب الاختبار . وبالنسبة للتربة التي تحتوى على طمي فإن اختبار الضاغط المتغير أكثر مناسبة إذ أن معدل التسرب المطلوب في الاختبار قليل مما يجعل مراقبة معدل انخفاض عمود المياه المتصل بالتربة داخل انبوبة رفيعة ، كما سيأتى ذكره في أسلوب الاختبار ، عمليا . أما بالنسبة للتربة الناعمة الطميية والطينية المشبعة فإن الأسلوب العملي للحصول على معامل نفاذيتها يكون بإجراء اختبار الانضغاط بالتصلب (كما سيأتى ذكره في الباب التاسع) .

6.4.1 Constant head permeability test

In the constant head permeability test, water is allowed to flow through a soil sample under a constant head (h). Knowing the length of the sample (L), and its cross sectional area (A), and measuring the discharge ($Q = V/t$) of the flowing water, the permeability coefficient (k) can be obtained from:

$$\frac{Q}{A} = k \cdot \frac{h}{L} = \frac{V}{t \cdot A}$$

$$k = \frac{V \cdot L}{h \cdot t \cdot A} \quad \dots \dots \dots (6.6)$$

The apparatus used in the laboratory is shown in Fig. 6.4. A soil sample is placed in a cylindrical container above a porous stone. The water is let in through a valve from a constant head tank, and the out let water is collected in a graduated cylinder. The difference in head is measured between the surface of the water topping the sample and that in a piezometric tube. The test is carried out several times with different values of (h) obtained by changing the position of the valve. The average value of (k) is calculated.

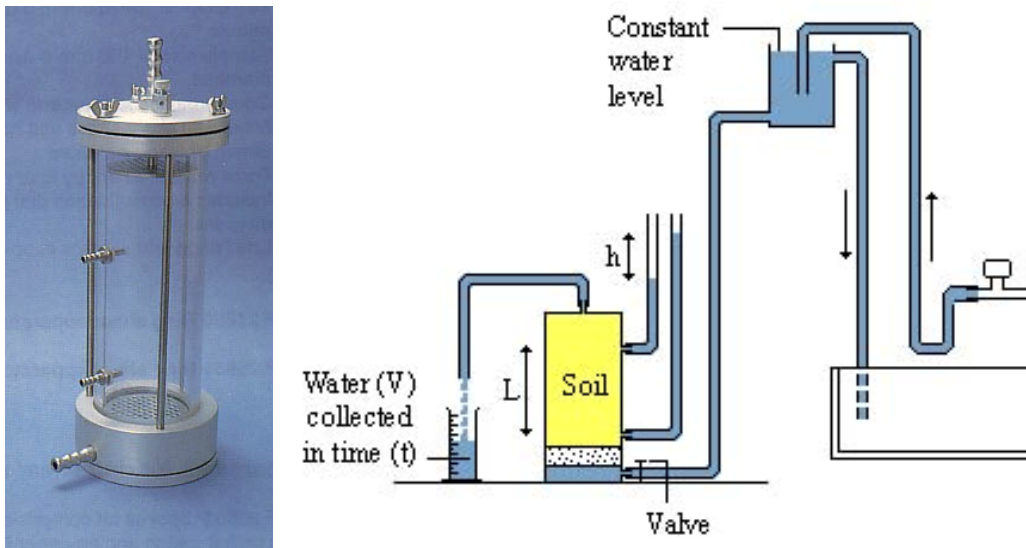


Fig. 6.4 constant head permeability test setup

6.4.2 Falling head permeability test

The constant head permeability test is used for coarse soils only where a reasonable discharge can be collected in a given time. However, the falling head test is used for relatively less permeable soils where the discharge is small. Fig 6.5 shows the used apparatus.

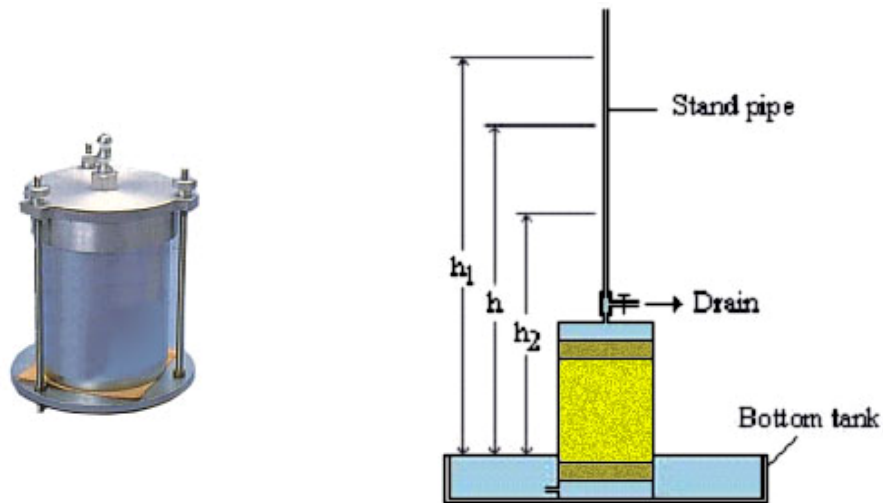


Fig. 6.5 Falling head permeability test setup

A stand pipe of known cross sectional area (a) is fitted over the permeameter, and water is allowed to run down. The water flows. Observations are started after steady state of flow has been reached. The head at any time instant (t) is equal to the difference in the water level in the stand pipe and the bottom tank.

Let (h_1) and (h_2) be the heads at time intervals (t_1) and (t_2), respectively. Let (h) be the head at any intermediate time interval (t) and ($-dh$) be the change in the head in a small time interval (dt). Hence from Darcy's law, the rate of flow (q) is given by:

$$q = \frac{-dh}{dt} \cdot a = k \cdot i \cdot A \quad \text{where:} \quad i = \frac{h}{L}$$

$$\therefore \frac{k h}{L} \cdot A = \frac{-dh}{dt} \cdot a$$

$$\text{or:} \quad \frac{K A}{a L} \cdot dt = \frac{-dh}{h} \quad \text{by integration:}$$

$$k = 2.3 \frac{a \cdot L}{A \cdot t} \log_{10} \frac{h_1}{h_2} \quad \dots \dots \dots (6.7)$$

6.5 Permeability of stratified soil

تتكون التربة في الطبيعة غالبا من طبقات مختلفة الخواص. لذلك فان تصرف المياه خلال التربة يعتمد على خواص تلك الطبقات وسمكها واتجاه سريان المياه خلالها. وسنتعرض هنا لمسألتين , ويلاحظ ان هذه الحالة تماثل محصلة تصرف المياه من مجموعة انابيب مختلفة الاقطار وتحت تأثير نفس الضاغط المائي . أما في الحالة الثانية تتحرك المياه في اتجاه عمودي على الطبقات , وهذه الحالة تماثل تحرك المياه في الماسورة يتغير قطرها كل طول معين منها وكذلك فان التصريف ثابت خلال الماسورة ولكن تتغير السرعة والفاقد من طول الى اخر حسب القطر.

In nature a soil deposit may consist of several layers, each layer has its value of permeability coefficient. The average permeability of the whole soil will depends on the direction of flow. Assuming that each soil layer is

horizontal and homogenous, two cases of flow will be considered, the first is in the horizontal direction, and the second is in the vertical direction.

6.5.1 Average permeability in the horizontal direction

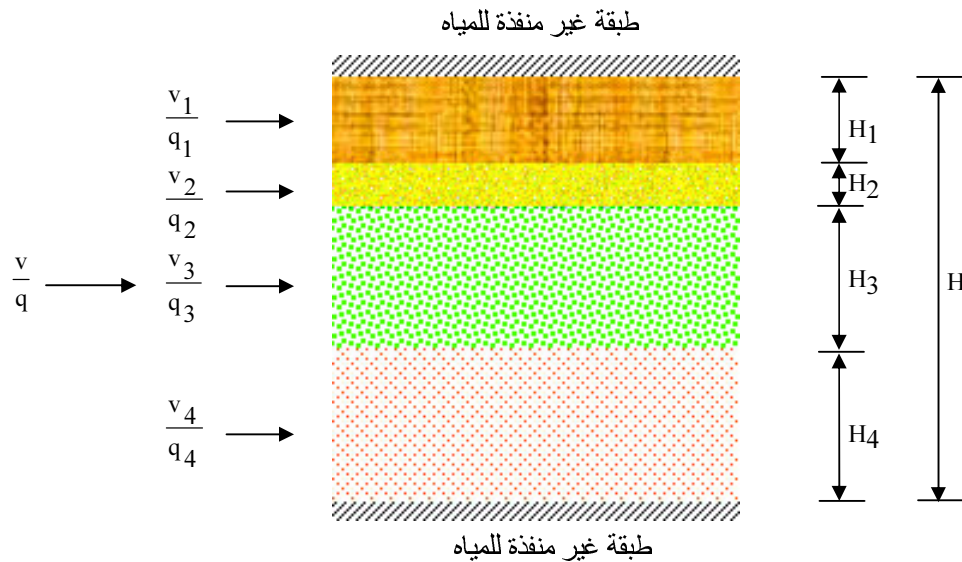


Fig. 6.6 Horizontal flow of water in stratified soil

Let $H_1, H_2 \dots$ etc. be the thickness of soil layers having permeability coefficient $k_1, k_2 \dots$ etc. For the flow in the horizontal direction, the hydraulic gradient (i) will be the same for all layers.

However, since $v = k \cdot i$, and since k is different, the velocity of flow will be different in different layers.

Let (k_I) be the average permeability of the soil in the horizontal direction. The total discharge through the soil equals the sum of discharges through the individual layers, i.e.:

$$q = k_I \cdot I \cdot H = k_1 \cdot i \cdot H_1 + k_2 \cdot i \cdot H_2 + k_3 \cdot i \cdot H_3 + \dots$$

Take: $H = H_1 + H_2 + H_3 + \dots$

Then:
$$k_I = \frac{k_1 H_1 + k_2 H_2 + k_3 H_3 + \dots}{H_1 + H_2 + H_3 + \dots} = \frac{\sum k H}{\sum H} \dots (6.8)$$

6.5.2 Average permeability in the vertical direction

In this case, the velocity of flow and hence the discharge will be the same through each layer but the hydraulic gradient will be different. Assuming that the head loss for the different layers is $h_1, h_2, h_3 \dots$ etc. and the total head loss is (h) , we have:

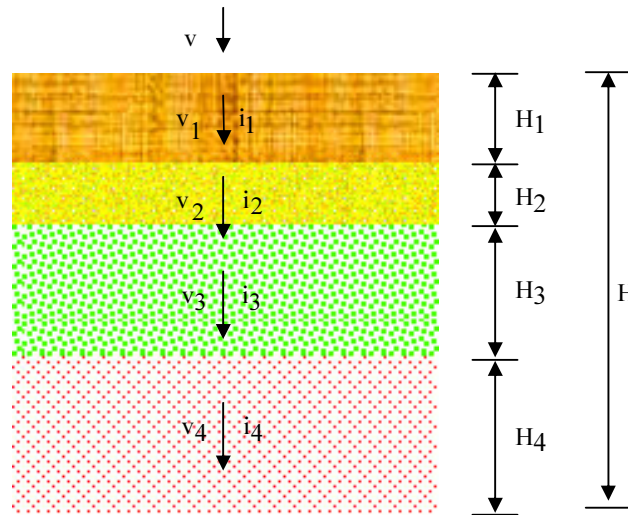


Fig. 6.7 Vertical flow of water in stratified soil

$$v = k_{II} \cdot i = k_{II} \cdot \frac{h}{\sum H}$$

$$h = \frac{v \cdot \sum H}{k_{II}} = h_1 + h_2 + h_3 + \dots$$

Also: $v = k_1 \frac{h_1}{H_1} = k_2 \frac{h_2}{H_2} = \dots$

$$\therefore \frac{v \cdot \sum H}{k_{II}} = v \frac{H_1}{k_1} + v \frac{H_2}{k_2} + v \frac{H_3}{k_3} + \dots$$

$$k_{II} = \frac{H_1 + H_2 + H_3 + \dots}{\frac{H_1}{k_1} + \frac{H_2}{k_2} + \frac{H_3}{k_3} + \dots} = \frac{\sum H}{\sum \frac{H}{K}} \dots (6.9)$$

٦.٦ Aquifers

تتكون الطبقات الحاملة للمياه الحرة من التربة الخشنة (رمل وزلط) عالية النفاذية مما يجعل حركة المياه خلالها سريعة والتصرف منها وفير . وللاستفادة من تلك المياه فى الزراعة أو الشرب تدق الآبار لتصل الى الطبقات الحاملة وتسحب منها المياه بالظلمبات . ومن أمثلة ذلك محطة مياه شمال القاهرة التى تحتوى على عدد كبير من الآبار العميقة تغذى جزء كبير من المدينة , وعشرات الآبار فى الاراضى الزراعية المستصلحة فى الطريق مصر – الاسماعيلية , مصر – الإسكندرية وأماكن أخرى عديدة .

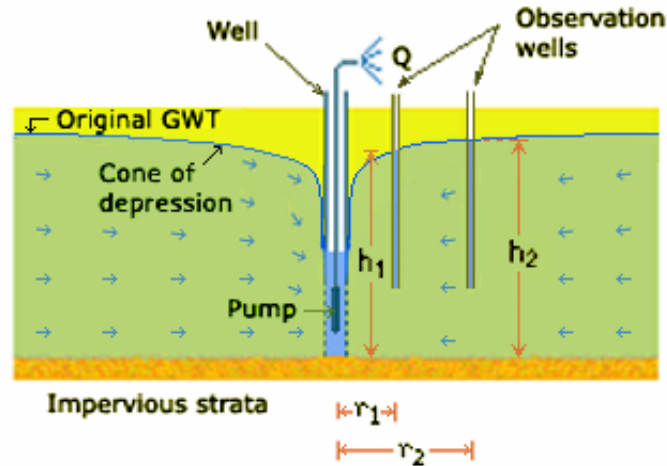
والجانب الآخر من التعامل مع الطبقات الحاملة للمياه الحرة يظهر عندما يمتد حفر لمشروع أسفل منسوب المياه الجوفية فتسحب المياه بالظلمبات من آبار خارج أو داخل الحفر مما يجعل سطح المياه الجوفية ينخفض ويتم العمل فى موقع جاف .

Aquifers are permeable formations having structures which permit appreciable quantity of water to move through them under ordinary conditions in the field. In order to use this groundwater, pumping is carried out from these aquifers. The discharge obtained depends on the permeability of the soil strata in which the well is sunk. Wells are also sunk in aquifers and water pumped out for the purpose of dewatering. Pumping out test is used for determining field permeability of soil formation.

6.6.1 Unconfined aquifer

When a well is penetrated into an aquifer, the water table initially remains horizontal. When the water is pumped from the well a curved depression in the water, called the cone of depression, occurs.

The water levels in the two observation walls (piezometers) at distances r_1 and r_2 are measured, Fig. 6.8. The relationship between the discharge and the coefficient of permeability can be obtained. Assuming that the flow is horizontal, and proportional to slope of the tangent to the top surface of groundwater table, at any radius (r):



$$Q = K \cdot i \cdot A$$

$$= k \cdot \frac{dh}{dr} \cdot 2 \pi r \cdot h$$

$$\int_{r_1}^{r_2} \frac{dr}{r} = \frac{2 \pi k}{Q} \int_{h_1}^{h_2} h \cdot dh$$

$$k = \frac{Q}{\pi} \cdot \frac{2.3 \log_{10} \left(\frac{r_2}{r_1} \right)}{(h_2)^2 - (h_1)^2} \quad \dots\dots\dots (6.10)$$

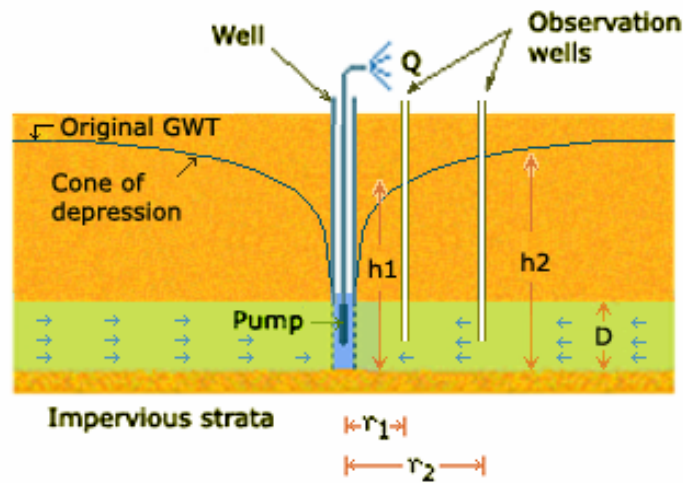
6.6.2 Confined aquifer

In this case the permeable stratum is overlaid by a layer of low permeability. From Fig. 6.9:

$$Q = K \cdot i \cdot A$$

$$= k \cdot \frac{dh}{dr} \cdot 2 \pi r \cdot D$$

$$\int_{r_1}^{r_2} \frac{dr}{r} = \frac{2 \pi k D}{Q} \int_{h_1}^{h_2} dh$$



$$k = \frac{Q}{2 \pi D} \cdot \frac{2.3 \log_{10} \left(\frac{r_2}{r_1} \right)}{h_2 - h_1} \quad \dots\dots\dots (6.8)$$

6.7 Seepage analysis

When water flows through a saturated soil mass, the total head at any point in the soil mass consists of:

- (a) piezometric head (pressure head)
- (b) velocity head
- (c) position head

The velocity head ($v^2 / 2g$) is negligibly small for flow of water through the soil. Hence, the total head at any point is equal to the algebraic sum of the piezometric head and the position head. In Fig. 6.10:

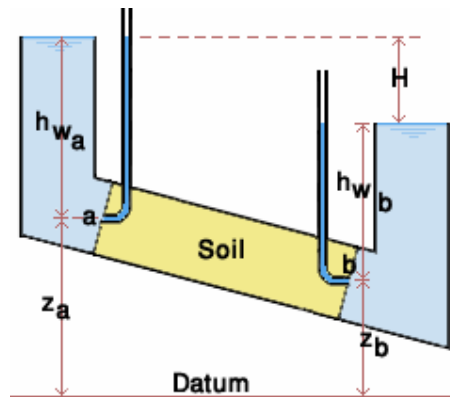


Fig. 6.10 Seepage of water through soil

Total head at (a) = $h_{wa} + z_a$ and

Total head at (b) = $h_{wb} + z_b$

Difference in total head for (a) and (b) = H

The difference (H) is also called the hydraulic head, and the loss of the hydraulic head per unit distance of flow through soil is called the hydraulic gradient.

6.7.1 Seepage pressure

الضغط الذي يسبب انسياب المياه خلال مسافة معينة في التربة يساوى فرق الضغوط خلال تلك المسافة مضروباً في الكثافة المياه ويكون اتجاهه في اتجاه سريان المياه . وهو ضغط فعال وزيادة قيمته تؤدي الى سحب الحبيبات التربة وتحركها من مكانها واذ تم ذلك راسياً لافى يقال ان التربة يحدث لها "تفوير" ولذلك فى الاعمال الى تتناول تحرك المياه خلال التربة يتم دراسة اتجاهات الانسياب والضغط المصاحبة منعا لافى اخطار او مشاكل.

Seepage pressure is the pressure exerted by water on the soil through which it percolates. It occurs due to the viscous friction between the water and soil particles. If (h) is the head loss due to the frictional drag of flowing water through a soil mass of thickness (z), the seeping pressure (P_s) is given by:

$$P_s = h \cdot \gamma_w$$

$$\text{or : } P_s = \left(\frac{h}{z}\right) \cdot z \cdot \gamma_w$$

The seepage acts in the direction of water flow. The vertical effective pressure may be increased or decreased due to the seepage pressure, depending on the direction of flow, i.e.:

In Fig. 6.11 the water flows in an upward direction through a saturated sample of thickness (z) under a hydraulic head (h):

Downward pressure = $z \cdot \gamma_{\text{sat}}$.

Upward pressure = $(h + z) \cdot \gamma_w$

At critical condition:

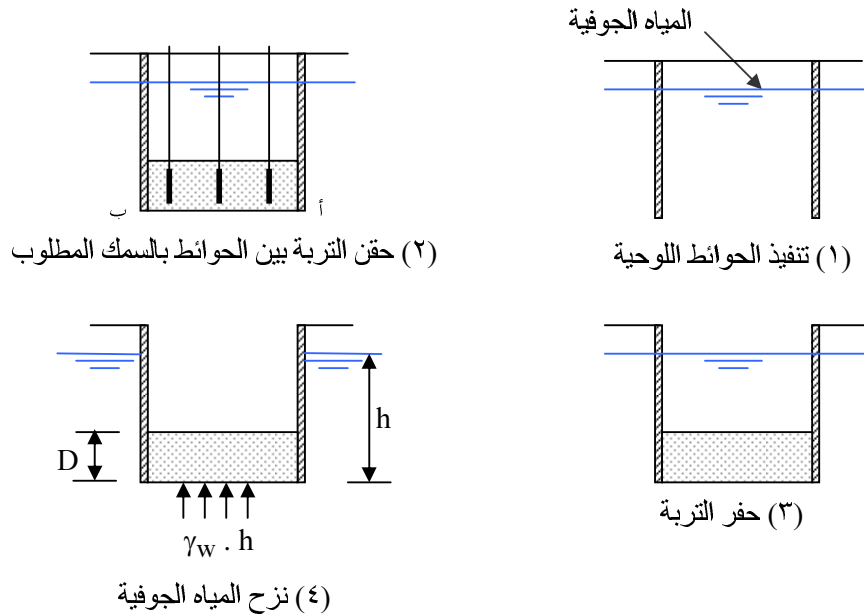
$$z \cdot \gamma_{\text{sat}} = (h + z) \cdot \gamma_w$$

$$= h \cdot \gamma_w + z \cdot \gamma_w$$

$$h \cdot \gamma_w = z \cdot (\gamma_{\text{sat}} - \gamma_w)$$

$$\therefore \frac{h}{z} = \frac{\gamma_{\text{sub.}}}{\gamma_w} = i_c$$

فى تنفيذ مترو الانفاق فى القاهرة فى المرحلة الاولى تم حقن المسافة (D) من التربة بحيث يصبح الضغط من اعلى الى اسفل على المنسوب (أ - ب) بتأثير وزن التربة المحقونة مكافئاً (على الاقل) لضغط المياه من اسفل الى اعلى $(\gamma_w \cdot h)$.



$$\sigma = z \cdot \gamma_{\text{sub.}} \pm P_s$$

... (6.12)

6.7.2 Upward flow (quick sand phenomenon)

In the case of upward flow, if the seepage pressure becomes equal to the pressure due to the submerged weight of the soil, the effective pressure is reduced to zero. In such case the soil loses all its strength and the soil particles will have a tendency to move upwards. This phenomenon is called quick sand or boiling condition. Thus, we have:

$$\sigma' = z \cdot \gamma_{\text{sub.}} - P_s = 0$$

$$P_s = z \cdot \gamma_{\text{sub.}}$$

$$i_c \cdot z \cdot \gamma_w = z \cdot \gamma_{\text{sub.}}$$

$$\therefore i_c = \frac{\gamma_{\text{sub.}}}{\gamma_w} \quad \dots \dots \dots (6.13)$$

where: i_c = critical hydraulic gradient.

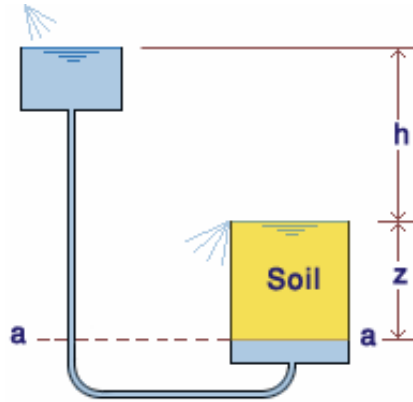


Fig. 6.11 Critical hydraulic gradient

6.7.3 Two dimensional flow انسياب المياه في الاتجاهين

Consider an element of soil, Fig. 6.12, of size Δx and Δy , and of unit thickness (perpendicular to the plane of the paper). The velocity of water entering the element, and the exit velocities will be as shown in the figure. Now, assuming that the soil and water are incompressible, the quantity of water flowing in and out of the element are equal. Assuming that Darcy's law is valid, we have:

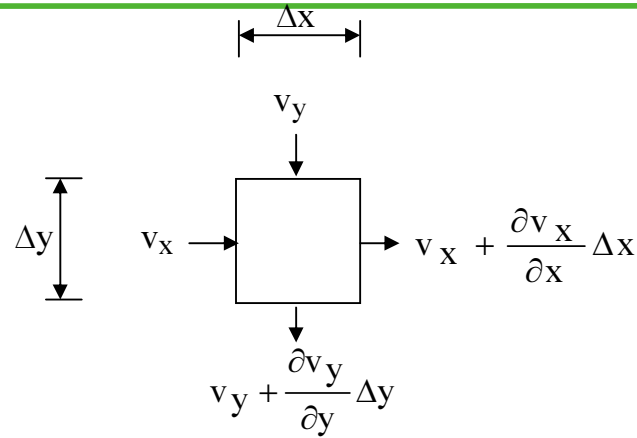


Fig. 6.12 Two-dimensional flow through soil element

$$v_x \cdot \Delta y + v_y \cdot \Delta x = (v_x + \frac{\partial v_x}{\partial x} \cdot \Delta x) \Delta y + (v_y + \frac{\partial v_y}{\partial y} \cdot \Delta y) \Delta x$$

$$\text{i.e. } \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \quad \dots \dots \dots (6.14)$$

Now, we have:

$$v_x = k_x \cdot i_x = k_x \cdot \frac{\partial h}{\partial x} \quad \&$$

$$v_y = k_y \cdot i_y = k_y \cdot \frac{\partial h}{\partial y}$$

where: k_x and k_y are the coefficients of permeability in the x and y directions, respectively. Substituting in Equ. (6.14):

$$\frac{\partial^2 (k_x \cdot h)}{\partial x^2} + \frac{\partial^2 (k_y \cdot h)}{\partial y^2} = 0$$

For isotropic soil : $k_x = k_y$

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0 \quad \dots \dots \dots (6.15)$$

Equation 6.15 is the **Laplace** equation of flow in two dimensions. The solution of the equation gives two sets of curves, known as equi-potential lines and flow lines. The equi-potential lines represent contours of equal head, and the flow lines gives the seepage direction which is perpendicular to the equi-potential lines. These can be obtained graphically as presented in the next section.

لدراسة اتجاهات انسياب المياه خلال التربة والتصرف الناتج والضغط المصاحبة لذلك يتم حل معادلة "لابلاس" عند نقطة مختلفة من التربة ويستدعى ذلك حل مئات من المعدلات الآنية . وتستخدم الطرق العددية والكمبيوتر بالبرامج الجاهزة في ايجاد الحلول لمسائل الانسياب اخذا في الاعتبار تجانس التربة او عدم تجانسها واختلاف معامل نفاذيتها في الاتجاهين الافقى والراسى وكافة المتغيرات المحتملة . والطرق التقليدية البسيطة المعروفة هي طريقة شبكة الانسياب التي اذا تم رسمها بمراعاة الشروط الاساسية للانسياب وبالدقة المناسبة اعطت تصورا مناسباً للحل .

6.7.4 Graphical method of flow net construction

The graphical method is based on trial sketching. The hydraulic boundary conditions have a great effect on the general shape of the flow net. The following conditions must be followed:

- (a) The flow lines and equal-potential lines meet at right angle to one another.
- (b) The fields are approximately square, Fig. 6.13, so that a circle can be drawn touching the four sides.
- (c) All curves should be smooth.
- (d) The number of flow channels is usually not less than 5.

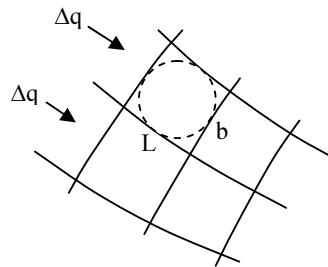


Fig. 6.13 Construction of the flow net

The following is an example of flow net construction:

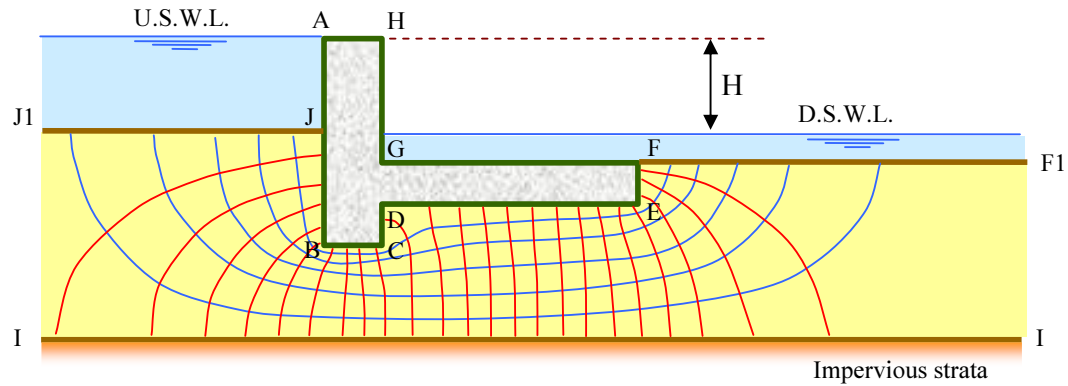


Fig. 6.14 Example of flow net

Fig 6.14 shows a flow net construction with a difference between upstream water level (USWL) and (DSWL) equal to (H). After several trials, the flow net is draw with flow channels and head drops. Notice that:

- (a) Lines JJ_1 and FF_1 are equi-potential lines.
- (b) Line ABCDEF is a flow line
- (c) Line I-I is a flow line
- (d) Flow lines and equi-potential lines are mostly perpendicular
- (e) Fields are mostly square
- (f) All lines are smooth.

6.7.5 Calculation of seepage discharge

In Fig 6.14, let:

Δh = head drop through the field

Δq = discharge passing through the flow channel

H = total pressure head.

Applying Darcy's law

$$q = k \cdot i \cdot A$$

$$\Delta q = k \cdot \frac{\Delta h}{L} \cdot (b \cdot 1) \quad \text{w here 1 is a linear dimension (1 متر مثلاً)}$$

Let: N_d = number of head drops
 N_f = number of flow channels

$$\Delta h = \frac{H}{N_d}$$

$$\Delta q = k \cdot \frac{H}{N_d} \cdot (b)$$

$$q = k \cdot \frac{H}{N_d} \cdot (b) \cdot N_f \quad \text{Taking } b = 1 \text{ (unit width)}$$

$$q = k \cdot H \cdot \frac{N_f}{N_d}$$

$$q = \text{التصرف للمتر الطولى} \dots \dots \dots (6.16)$$

In the previous example:

$$N_f = 6$$

$$N_d = 24$$

Taking for example: $k = 1 \times 10^{-3}$ cm/sec, and $H = 3$ m:

$$q = \frac{1 \times 10^{-3}}{10} \times 60 \times 60 \times 24 \times 3 \times \frac{6}{24} = 0.648 \text{ m}^3 / \text{day} / \text{m}'$$

6.7.6 Calculation of seepage pressure

The hydraulic potential (h) at any point located after (n) head drops, each drop of value (Δh), is given by:

$$h = H - n \cdot \Delta h$$

$$p_s = h \cdot \gamma_w = (H - n \cdot \Delta h) \gamma_w$$

From Fig. 6.15 the seepage pressure on section a-b-b1 is:

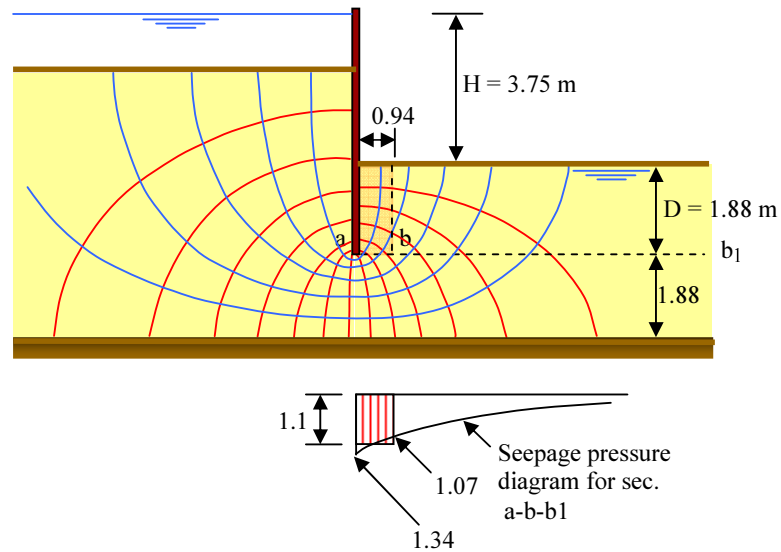


Fig. 6.15 Check of downstream stability against piping

$$p_{s1} = 3.75 - 9 \times 3.75 \div 14 = 1.34 \text{ t/m}^2$$

$$p_{s2} = 3.75 - 10 \times 3.75 \div 14 = 1.07$$

From these values the seepage pressure diagram can be drawn. In order to check the stability of the downstream against piping consider the forces acting on the line a-b, where $ab = D/2 = 1.88/2 = 0.94 \text{ m}$. The average seepage force is:

$$\text{Force} \uparrow = 1.1 \times 0.94 \times 1.0 = 1.03 \text{ t/m'}$$

The downward force is calculated, assuming that the saturated density of the soil is 1.8 t/m^3 , as:

$$\text{Force} \downarrow = 1.88 \times 0.94 \times 0.8 = 1.41 \text{ t/m'}$$

$$\text{The factor of safety against piping (F.S.)} = 1.41 / 1.03 = 1.37$$

في كثير من الاحيان ولظروف تكوين التربة وترسيبها يكون معامل نفاذية التربة في الاتجاه الافقى مختلفا عن معامل النفاذية في الاتجاه الرأسى . و احيانا تكون المنطقة الحاملة للمياه الجوفية مكونة من الطبقات افقية متجانسة افقيا وراسيا , لكن لكل طبقة منها معامل نفاذية مختلف . وهذه الحالات موجودة بكثرة في الطبيعة وفيما يلي ملخصا لكيفية التحليل لكل حالة .

6.7.7 Seepage through anisotropic soil

If the horizontal and vertical coefficients of permeability of a soil are different, i.e. $k_x \neq k_y$ transformation is carried out by putting:

$$x_n = x \sqrt{\frac{k_y}{k_x}} \quad \dots \dots \dots (6.17)$$

According to this transformation, the Laplace's equation becomes:

$$\frac{\delta^2 h}{\delta x_n^2} + \frac{\delta^2 h}{\delta y_n^2} = 0 \quad \dots \dots \dots (6.18)$$

To draw the flow net for an anisotropic soil:

- (a) The cross-section of the structure is redrawn such that all dimensions parallel to the x-axis are transformed by multiplying by $\sqrt{k_x/k_y}$:
- (b) A transformed flow net is drawn on the new drawing following same rules as mentioned before.
- (c) The actual flow net can be obtained by re-transforming the section and flow net back to actual dimensions, Fig.6.16.
- (d) The discharge is calculated from the relation:

$$q / m' = k_{\text{equivalent}} \cdot H \cdot \frac{n_f}{n_d} \quad \dots \dots \dots (6.19)$$

where:

$$k_{\text{equivalent}} = \sqrt{k_x \cdot k_y}$$

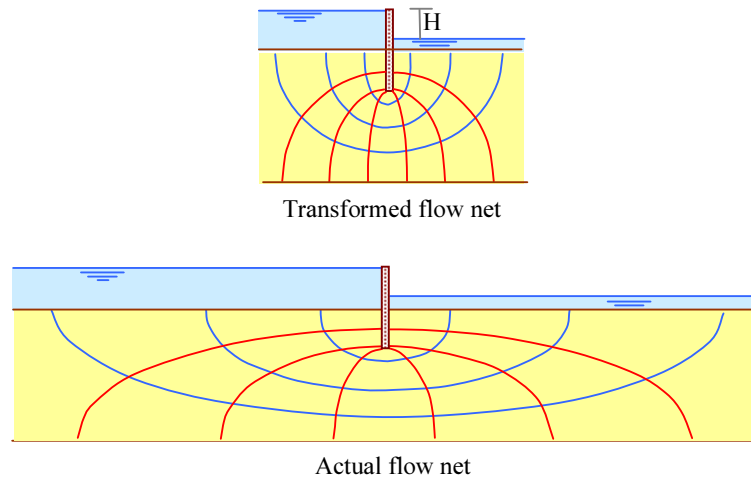


Fig. 6.16 Flow net for anisotropic soil

6.7.8 Seepage through layered soil

In case the permeable strata is formed of isotropic layers having different permeability coefficients, the flow lines deflect at the interfaces of these layers. Fig.6.17 shows an example of this case. Equating the discharge in a flow channel within soil (1) with that in the corresponding flow channel in soil (2), the following relation can be obtained:

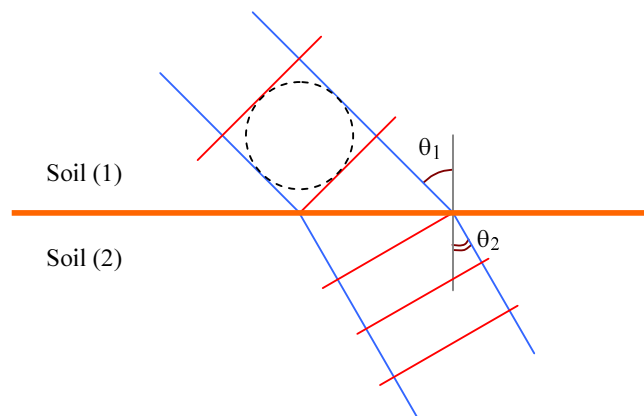


Fig. 6.17 Deflected flow lines in layered soil

$$\frac{\tan \theta_1}{\tan \theta_2} = \frac{k_1}{k_2} \quad \dots \dots \dots (6.20)$$

6.7.9 Protective filters

تستخدم الفلاتر - التي تتكون من التربة المتدرجة - في تنظيم حركة المياه من التربة الى داخل مصافي الابار ، وفي وقاية التربة عند مخارج المياه المناسبة اسفل المنشآت مثل الستائر المعدنية والهدارات والخزانات الترابية. ويجب ان يسمح الفلتر للمياه للتصرف خلاله بسلاسة كافية على الا يسمح لحبيبات التربة بالمرور خلاله تحت تأثير ضغط الانسياب. وغالبا ما يتكون الفلتر من الرمل الخشن والزلط الرفيع او الزلط المتدرج حسب تدرج التربة المستخدم فيها.

A protective filter consists of a combination of layers of pervious materials. It is designed in such a manner as to provide the following:

- (a) Quick drainage of water
- (b) Prevention of movement of soil particles due to flowing water.
- (c) The filter thickness should ensure adequate factor of safety against piping.

It was found from practice that the filter gradation with respect to the soil is as shown in Fig. 6.18.

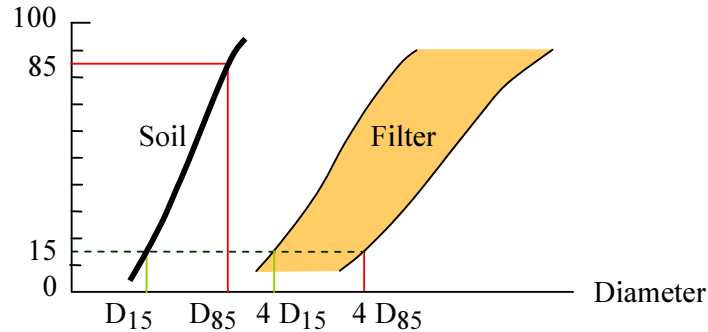


Fig. 6.18 Design of filter

6.8 Examples

(1) A constant head permeability test is carried out on a silty sand soil sample. The cross sectional of the sample is 50 cm^2 and its height is 25 cm. Results are as follows:

h cm	t min.	V cu.cm
5	60	82
10	60	180
15	60	235
20	15	86
25	15	110
30	15	150

Required :

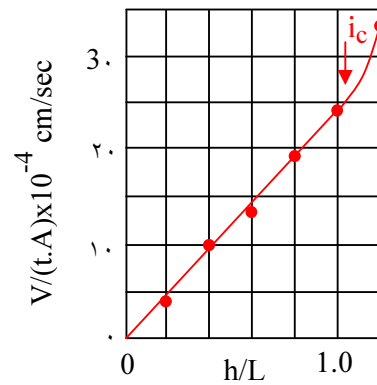
(a) Draw the relation between h/L (as abscissa) and $V/(t.A)$ (as ordinate). From the relation determine the average coefficient of permeability of the sand.

(b) Knowing the dry density of the tested soil is 1.66 t/m^3 , and the specific gravity is 2.65, find the critical hydraulic gradient.

(c) Find the seepage velocity at a hydraulic gradient of 0.7.

Solution:

h/L	$V/(t.L) \times 10^{-4} \text{ cm/sec}$
0.2	4.56
0.4	10.0
0.6	13.06
0.8	19.11
1.0	24.44
1.2	33.33



(a) From curve:

$$\text{Average slope} = k = 0.0023 \text{ cm/sec}$$

$$(b) \quad \gamma_d = \frac{G_s}{1+e} \gamma_w$$

$$1.66 = \frac{2.65}{1+e} \times 1$$

$$e = 0.596$$

$$i_c = \frac{\gamma_{\text{sub.}}}{\gamma_w} = \frac{G_s - 1}{1+e}$$

$$= \frac{2.65 - 1}{1 + 0.596} = 1.03$$

$$(c) \quad n = \frac{e}{1+e} = \frac{0.596}{1 + 0.596} = 0.374$$

$$v = k \cdot i$$

$$= 0.0023 \times 0.7 = 0.00161 \text{ cm / sec}$$

$$v_s = \frac{v}{n} = \frac{0.00161}{0.374} = 0.0043 \text{ cm / sec}$$

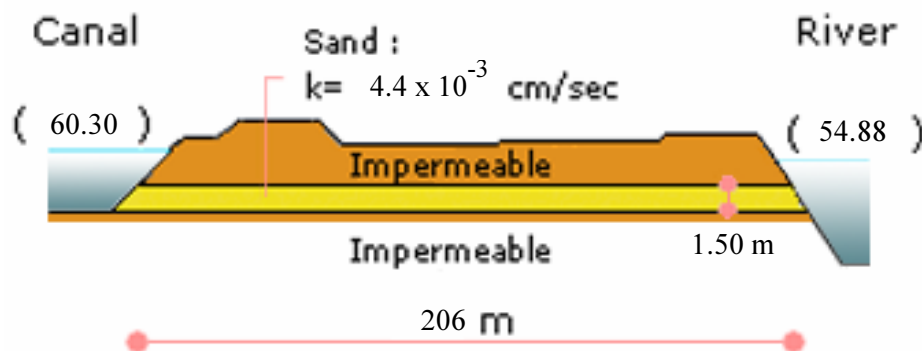
(2) In a falling head permeability test the initial head was 40 cm. After 10 min. the head dropped 5 cm. Calculate the soil's coefficient of permeability. Also, calculate the time for the head to drop another 15 cm. Take length of sample 6 cm, area of sample 50 cm^2 and area of stand pipe 0.5 cm^2 .

Solution:

$$\begin{aligned} \text{(a) } k &= 2.3 \frac{a \cdot L}{A \cdot t} \log_{10} \frac{h_1}{h_2} \\ &= 2.3 \frac{0.5 \times 6}{50 \times (10 \times 60)} \log_{10} \frac{40}{35} = 1.33 \times 10^{-5} \text{ cm/sec} \end{aligned}$$

$$\begin{aligned} \text{(b) } 1.33 \times 10^{-5} &= 2.3 \frac{0.5 \times 6}{50 \times t} \log_{10} \frac{35}{20} \\ \therefore t &= 2521.7 \text{ sec} \approx 42 \text{ min} \end{aligned}$$

(3) The figure shows a canal aligned parallel to a river. The soil formation includes a permeable layer intercepting the two water ways. Calculate the seepage from the canal to the river in $\text{m}^3/\text{day}/\text{km}$.



Solution:

$$\begin{aligned} q &= k \cdot i \cdot A \\ &= \frac{4.4 \times 10^{-3}}{100} \times \frac{(60.3 - 54.88)}{206} \times 1.5 \times 1000 \times 60 \times 60 \times 24 \\ &= 150 \text{ m}^3/\text{day}/\text{km} \end{aligned}$$

(4) A clay deposits contains silty sand laminations at average vertical spacing of 1.2 m. The average thickness of lamination is 5 mm. Assuming that the coefficient of permeability of the silty sand is 100 times that of the clay, calculate the ratio between the horizontal and vertical permeability of the soil.

Solution:

Let : n = number of layers of clay and laminations

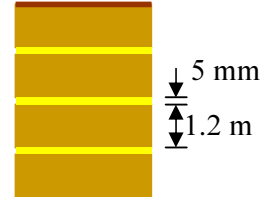
k_c = coefficient of permeability of the clay

$\therefore$ Coefficient of permeability of silt = $100 k_c$

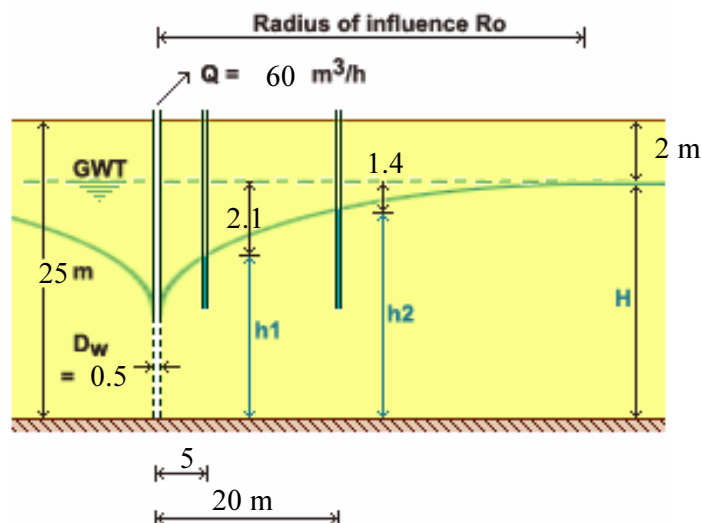
$$k_I = \frac{\sum H \cdot k}{\sum H} = \frac{(1.2 k_c + 0.005 \times 100 k_c) \cdot n}{(1.2 + 0.005) \cdot n} = 1.416 k_c$$

$$k_{II} = \frac{\sum H}{\sum \frac{H}{k}} = \frac{(1.2 + 0.005) \cdot n}{\left(\frac{1.2}{k_c} + \frac{0.005}{100 k_c}\right) \cdot n} = 1.004 k_c$$

$$\therefore \frac{k_I}{k_{II}} = \frac{1.416 k_c}{1.004 k_c} = 1.41$$



(5) The data for a pumping well test in an unconfined aquifer are as shown. Find the coefficient of permeability of the aquifer. Knowing that the original groundwater table is at 2 m depth, and the diameter of the well is 0.5 m, find the radius of influence of the well, and the draw down at the well, both corresponding to the given discharge.



Solution:

$$\begin{aligned}
 \text{(a) } k &= \frac{Q}{\pi} \frac{2.3 \log_{10} \left(\frac{r_2}{r_1} \right)}{h_2^2 - h_1^2} \\
 &= \frac{60 \times 1000000}{\pi \times 60 \times 60} \times \frac{2.3 \log_{10} \left(\frac{20}{5} \right)}{(2160)^2 - (2090)^2} \\
 &= 0.0247 \text{ cm/sec}
 \end{aligned}$$

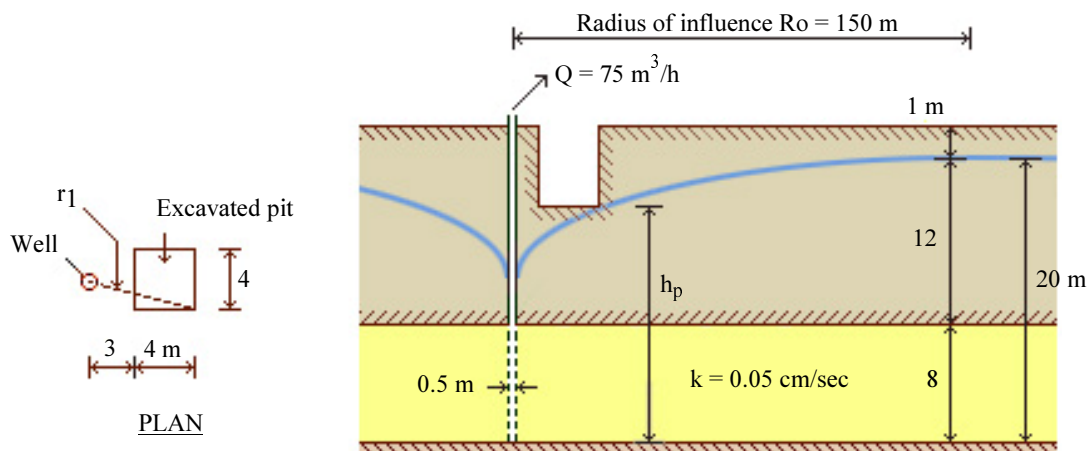
(b)

$$0.0247 = \frac{60 \times 1000000}{\pi \times 60 \times 60} \times \frac{2.3 \log_{10} \left(\frac{R_o}{20} \right)}{(2300)^2 - (2160)^2} \quad \therefore R_o = 367 \text{ m}$$

$$0.0247 = \frac{60 \times 1000000}{\pi \times 60 \times 60} \times \frac{2.3 \log_{10} \left(\frac{20}{0.25} \right)}{(2160)^2 - (h_o)^2} \quad \therefore h_o = 19.3 \text{ m}$$

 $\therefore$ Drawdown at the well = $23 - 19.3 = 3.7 \text{ m}$

(6) A foundation pit is excavated at a site where a nearby deep well already exists. The well and soil strata are shown in the figure. Calculate the maximum depth of excavation so that lowered groundwater level will not be above the bottom level of the excavation.



Solution:

$$\text{Maximum distance from well to excavation} = \sqrt{(7)^2 + (2)^2}$$

$$k = \frac{Q}{2\pi D} \cdot \frac{2.3 \log_{10}\left(\frac{r_2}{r_1}\right)}{h_2 - h_1}$$

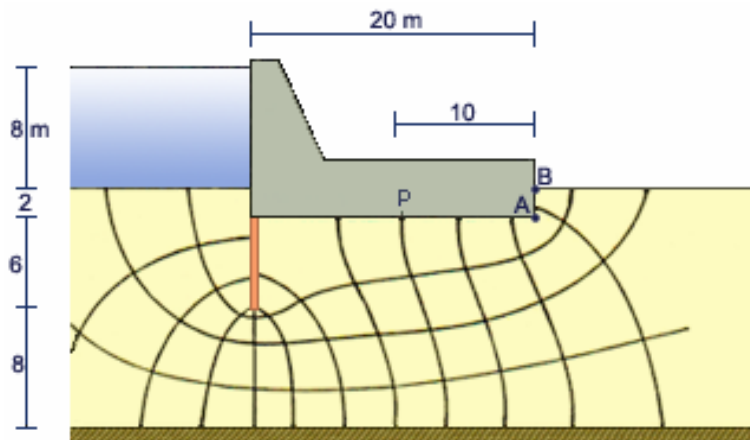
$$0.05 = \frac{75 \times 1000000}{60 \times 60 \times 2\pi \times 800} \cdot \frac{2.3 \log_{10}\left(\frac{150}{7.28}\right)}{2000 - h_p}$$

$$\therefore h_p = 17.5 \text{ m}$$

$$\therefore \text{Maximum depth of excavation} = 20 - 17.5 + 1 = 3.5 \text{ m}$$

(7) A concrete weir with a steel sheet pile, 6 m deep at the heel, is built at a site where the soil consists of sand strata of depth 16 m. The length of the base of the weir is 20 m, founded at 2 m below ground surface. The weir is to retain 8 m of water in the up stream. The soil has a coefficient of permeability of 4.6×10^{-4} cm/sec, specific gravity of 2.65, and void ratio of 0.72.

- Draw the flow net and calculate the amount of water seeping under the weir per 1 m width.
- Find the seepage force per 1 m³ at the toe of the foundation and check piping.
- Draw the uplift pressure diagram on the bottom of the weir.
- Find the seepage velocity at the toe.



Solution:

(a) From figure : $N_f = 4$, $N_d = 12$

$$\therefore q = k \cdot H \cdot \frac{N_f}{N_d} = \frac{4.6 \times 10^{-4}}{100} \times \frac{4}{12} \times 8 = 12.27 \times 10^{-6} \text{ m}^3/\text{sec}/\text{m}'$$

(b) Head drop in each "square" element $= \frac{8}{12} = 0.667 \text{ m}$

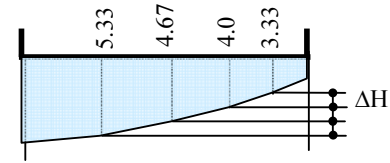
Average length of side of square element at toe $= 1.5 \text{ m}$

$$\therefore i = \frac{\Delta H}{L} = \frac{0.667}{1.5} = 0.444$$

$$i_c = \frac{G_s - 1}{1 + e} = \frac{2.65 - 1}{1 + 0.72} = 0.96$$

$\therefore i < i_c$ i.e. no piping will occur.

$$\text{Seepage force}/1 \text{ m}^3 = i \cdot \gamma_w = 0.444 \times 1 = 0.444 \text{ t}$$



(c) At each point of intersection of equipotential line with the bottom line of the weir, calculate the head loss, then the total head. For example at point P :

$$\text{Total head loss} = 8 \times 0.667 = 5.33 \text{ m}$$

$$\text{Head at point P} = 10 - 5.33 = 4.67 \text{ m}$$

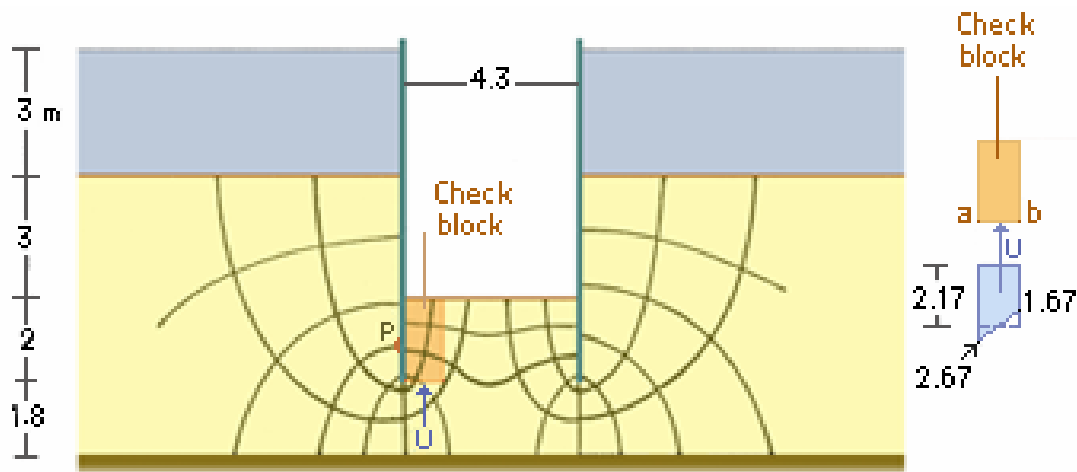
$$\therefore \text{Uplift at P} = h \cdot \gamma_w = 4.67 \times 1 = 4.67 \text{ t/m}^2$$

$$(d) v = k \cdot i = 4.6 \times 10^{-4} \times 0.44 = 2.04 \times 10^{-4} \text{ cm/sec}$$

$$v_s = \frac{v}{n} \quad n = \frac{e}{1 + e} = \frac{0.72}{1 + 0.72} = 0.418$$

$$v_s = \frac{2.04 \times 10^{-4}}{0.418} = 4.88 \times 10^{-4} \text{ cm/sec}$$

(8) The foundation of a bridge pier is constructed within a cofferdam formed of two sheet piles. The width of the cofferdam is 4.6 m; the water depth in the water way is 3 m, and the foundation depth is 3 m. The soil is formed of sand to 6.8 m below bed level, underlaid by impervious strata. The permeability of the sand is 1.2×10^{-5} m/sec, the void ratio is 0.5 and the specific gravity is 2.6. Draw the flow net and calculate the quantity of seepage in the cofferdam. Calculate the water pressure at a point 1 m below foundation level at upstream face of the sheet pile. Check safety against piping.



(a) From the figure: $N_f = 5$ $N_d = 9$

$$q = k \cdot H \cdot \frac{N_f}{N_d} = 1.2 \times 10^{-5} \times \frac{5}{9} \times 6 = 3.96 \times 10^{-5} \text{ m}^3/\text{sec}/\text{m}'$$

(b) To find water pressure at point P:

$$\Delta H = \frac{6}{9} = 0.667 \quad \therefore \text{Head loss at point P} = 0.667 \times 3 = 2 \text{ m}$$

$$\therefore \text{Water pressure at P} = 7 - 2 = 5 \text{ t/m}^2$$

$$(c) \text{ From the figure: } U = 2.2 \text{ t/m}^2 \quad \& \quad \gamma_{\text{sat.}} = \frac{2.6 + 0.5}{1 + 0.5} = 2.07 \text{ t/m}^3$$

$$\therefore W = 2 \times 1 \times 1.07 = 2.14 \text{ t/m}^2 \quad \therefore \text{F.S.} = \frac{2.14}{2.2} = 0.97 !!$$

6.9 Problems

(1) Results of a constant head permeability test are as follows:

- Volume of collected water = 1.1 lit.
- Diameter of sample = 8 cm
- Length of sample = 20 cm
- Head difference = 25 cm
- Time of collection of water = 5 min.

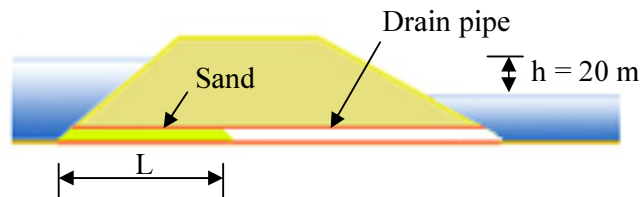
Calculate the coefficient of permeability in m/sec.

(2) A laboratory falling head permeability test is carried out on a soil sample. Results of the test are as follows:

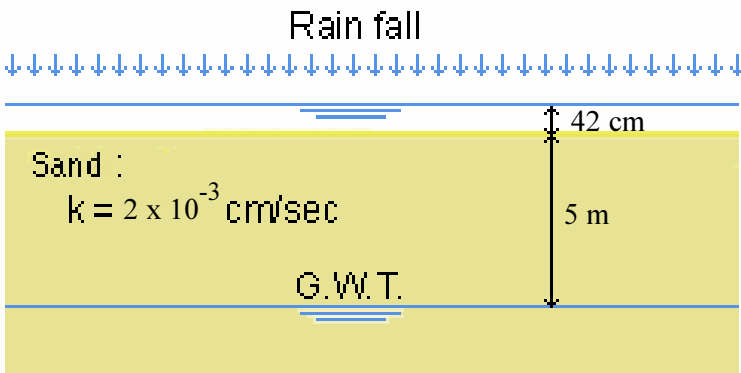
Diameter of sample = 8 cm
 Length of sample = 20 cm
 Diameter of stand pipe = 1.1 cm
 Initial head = 80 cm
 Final head = 40 cm
 Time of fall = 0.8 hour

Calculate the coefficient of permeability in m/sec.

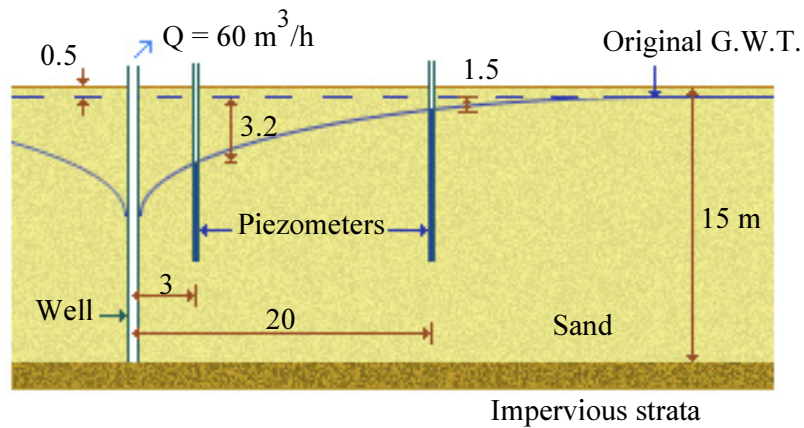
(3) A drain pipe 30 cm diameter lies beneath an earth dam. The pipe was partially clogged with sand. The discharge of the pipe was 300 lit/day when the difference between the dam's up stream and down stream water levels was 20 m. If the permeability of the sand was estimated to be 0.01 cm/sec, what would be the clogged length.



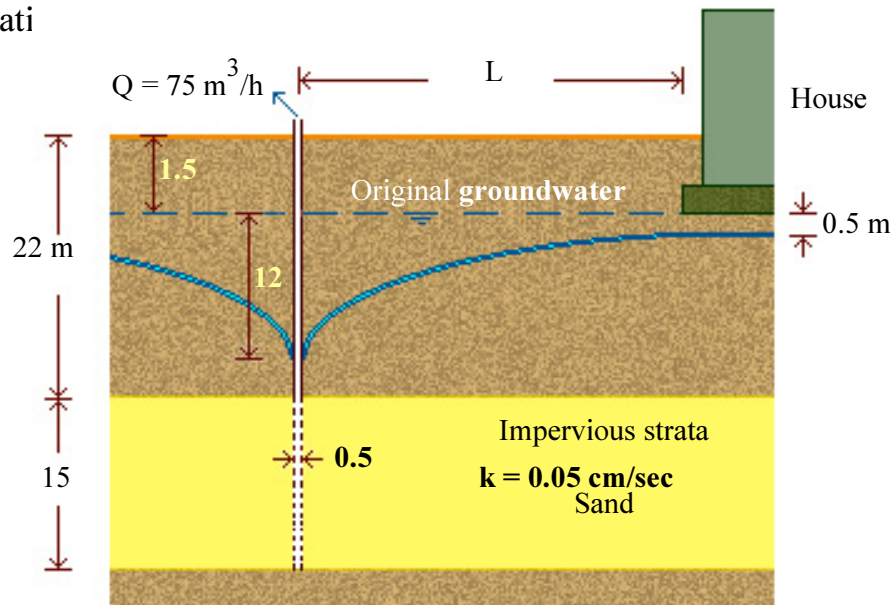
(4) Due to heavy rain fall, 42 cm of water collected above ground surface. Calculate the time for the rain water to infiltrate into the soil and disappears below ground surface. Use the data shown in figure



- (5) A soil formation consists of three layers 3, 7, 19 m thick. The coefficients of permeability of the layers are 6×10^{-4} , 1×10^{-3} and 1.1×10^{-2} cm/sec, respectively. Find the average permeability of the formation in the horizontal and vertical directions .
- (6) A bed of sand consists of three layers of equal thickness. The coefficients of permeability of the top and bottom layers are 2×10^{-4} cm/sec, and that of the middle layer 8×10^{-3} cm/sec. Find the ratio between the average horizontal and average vertical permeabilities of the sand bed.
- (7) A silt soil sample 5 cm diameter and 18 cm long was tested in a falling head permeameter. The time elapsed for the head to drop from 40 to 25 cm is 2.2 h. The stand pipe has a cross sectional area of 2 cm^2 . After test, the sample was splitted (شطرت) and was found to include a sandy silt lamination 0.5 cm thick. Knowing that the coefficient of permeability of the silt, as determined from other tests, is 8×10^{-5} cm/sec, determine the permeability of the laminations.
- (8) A puping test is carried out in the shown formation. Data are as given in figure. Calculate the permeability of the aquifer in m/day. Calculate the influence radius corresponding to the given discharge.



(9) A deep well is constructed at a site where the ground conditions are as shown in figure. The maximum discharge of the well is $30 \text{ m}^3/\text{h}$, giving a draw down of 12 m at the well. Calculate the minimum distance (L) from the well to a near-by house so that groundwater lowering under the foundation

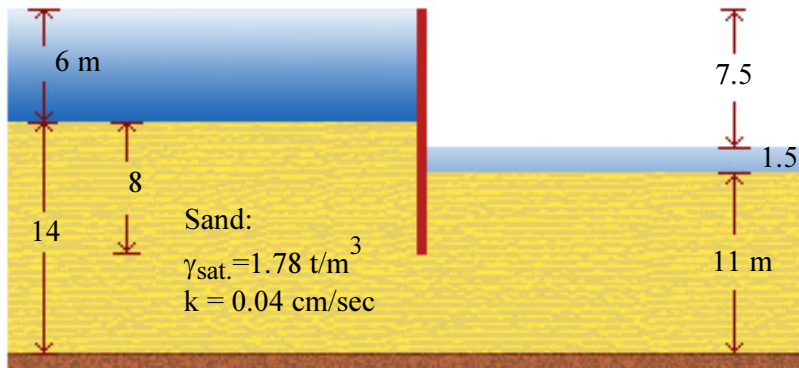


Impervious strata

(10) For the shown sheet pile:

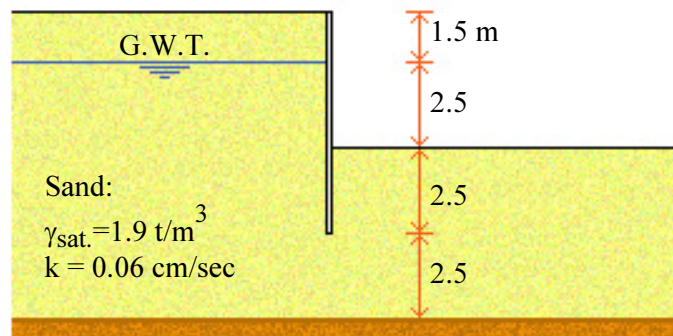
- Draw the flow net.
- Find the seepage of discharge in $\text{m}^3/\text{day}/\text{m}'$

- (c) Calculate the water pressure at the bottom tip of the sheet pile.
 (d) Calculate the water pressure at the bottom tip of the sheet pile.

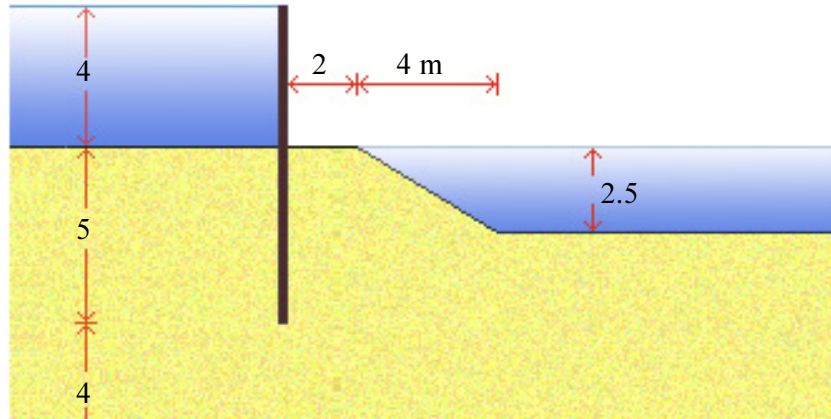


(11) For the shown sheet pile:

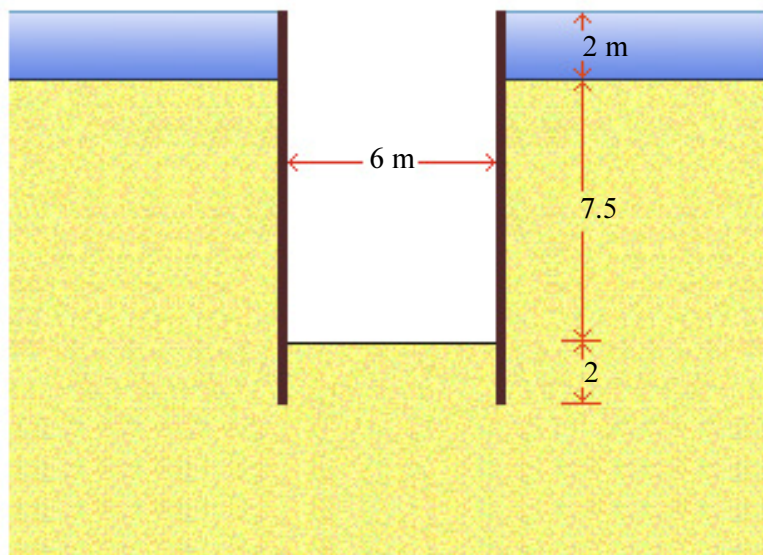
- (a) Draw the flow net.
 (b) Find the seepage of discharge in $\text{m}^3 / \text{day}/\text{m}'$.
 (c) Calculate the factor of safety against piping.
 (d) Draw the water pressure diagram on the up stream and down stream faces of the sheet pile.



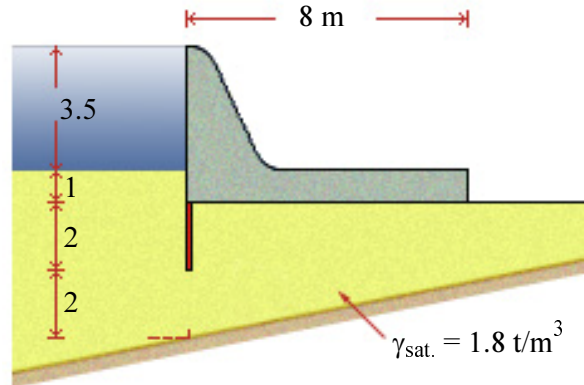
- (12) Sketch the flow net for seepage under the shown sheet piling. Find the quantity of seepage discharge knowing that the permeability of the sand is 0.0054 cm/sec .



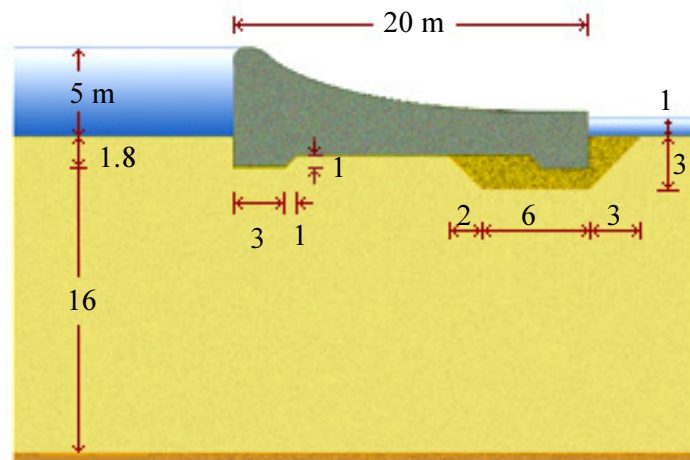
(13) For the shown cofferdam, sketch the flow net. Calculate the seepage discharge. Find the factor of safety against piping assuming that the saturated density of the soil is 1.88 t/m^3 . Take $k = 0.0054 \text{ m/sec}$.



(14) For the shown weir with sheet pile, draw the flow net and calculate the amount of water seeping under the weir per 1 m of its width. Find the seepage force per 1 m³ at the toe of the foundation and check piping. Draw the uplift pressure diagram on the bottom of the weir. Take $k = 0.0008$ m/sec.



(15) For the shown concrete dam, sketch the flow net. Calculate the amount of water seeping under the dam per 1 m width. Take $k = 0.00032$ m/sec.



(16) For the shown retaining wall sketch the flow net. Calculate the seepage discharge. Draw water pressure diagrams on planes AB and CD. Draw the uplift diagram on base BC. Take $k = 0.072 \text{ cm/sec}$.

